Inference From a Pictograph: Statistical Literacy in Action

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Pictographs are often used in the media to draw attention to data that would likely be ignored in a table. In school, however, pictographs disappear from the curriculum by the middle primary years. The outcomes of the research reported here indicate that pictographs can provide a basis for rich tasks displaying not only students' counting skills but also their appreciation of variation and uncertainty in prediction. The range of responses is discussed in relation to other research and classroom implications.

The use of pictographs in the early childhood/primary mathematics classroom is a common introduction to statistics in school. Pictographs have been shown to be useful for the development of basic counting and graph-reading skills embedded in a context easily recognisable to young children (Neal, 1994). This observation supports curriculum documents that have included pictographs in recommendations for the development of graphing skills at an introductory level. In particular, the Australian Education Council (AEC) in *A National Statement on Mathematics for Australian Schools* (1991) suggests that:

Initially, there should be an emphasis on the use of actual objects or physical representations of objects or measurements, although children should gradually move to simple pictorial, block and bar graphs. Thus students may construct concrete or pictorial graphs to compare collections by one-to-one correspondence. (p. 165)

In corroboration of this view, pictographs are mentioned explicitly in *Mathematics – A Curriculum Profile for Australian Schools* (AEC, 1994) as part of educational outcomes for chance and data in Level 1 (1.26 & 1.27), Level 2 (2.26), and Level 3 (3.26). Although recommending pictographs as an introductory measure to developing graphing techniques, in both documents the explicit reference to pictographs ends here. The absence of reference to pictographs in the higher levels reinforces the belief that students outgrow the applicability of pictographs as they move through the grade levels. A similar view appears to be held by the National Council of Teachers of Mathematics (NCTM) (2000).

A fundamental idea in prekindergarten through grade 2 is that data can be organized or ordered and that this "picture" of the data provides information about the phenomenon or question. In grades 3-5, students should develop their skills in representing their data, often using bar graphs, tables, or line plots ... (p. 49)

Turning from early childhood to adulthood, however, pictographs recur in the media (Pereira-Mendoza, Watson, & Moritz, 1995), as seen for example in Figure 1. In adulthood, the interpretation of such representations comes under the banner of adult statistical literacy. As outlined by Gal (2002) the important components of statistical literacy for adults are:

... (a) people's ability to *interpret and critically evaluate* statistical information, data-related arguments, or stochastic phenomena, which they may encounter in diverse contexts, and when relevant (b) their ability to *discuss or communicate* their reactions to such statistical information, such as their understanding of the meaning of the information, their opinions about the implications of this information, or their concerns regarding the acceptability of given conclusions. (pp. 2-3)

Because pictographs provide statistical information and often data-related arguments they certainly qualify as objects for interpretation, for critical evaluation, and for discussion or communication of the conclusions presented. Their prominence in the media further enhances the importance of a statistically literate public to interpret appropriately the information they portray. Since this is true for adults, why stop considering pictographs from the primary grades? They usually provide a straightforward context without complications and hence it is possible to use them to focus on interpretation and prediction. Starting early can assist in building the communication skills used later when contexts and representations are more complex.

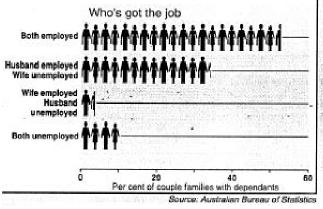


Figure 1. Example of a pictograph from the media (*The Australian*, 10/11/94, p.3).

Pereira-Mendoza and Mellor (1991), working with a variety of graph types, found that grade 4 and 6 children had a firm grasp on basic graph reading, however there were difficulties when students were required to provide an interpretation or make a prediction. Problems in interpretation were based mainly on computational errors, reading/ language misinterpretation, or scale errors. For predictions, difficulty was generally associated with

interference of topic knowledge, scale, patterns in data arrangement, or the fact that information asked for by the researchers was not shown explicitly on the graph. Watson and Moritz (2001) used similar ideas and prompts in an interview setting where students created their own pictographs. There was an increasing facility with grade in students' ability to answer questions that required interpretation and prediction, contrasting with the results of Pereira-Mendoza and Mellor who found little improvement for questions requiring prediction. In a study with six-year-olds, Watson and Kelly (2002) used two protocols relevant to the current study, one using pictographs adapted from Watson and Moritz and the other using a bar graph in a transport context adapted from Watson and Moritz (1999). Watson and Kelly found that children as young as six could observe and represent quite well, however, most students experienced difficulty in predicting and relied on idiosyncratic ideas, including a refusal to guess, personal experiences, and story telling.

Three research questions are suggested by the diminished recognition of pictographs across the curriculum, yet their potential to require interpretation in the adult media.

- 1. In what ways can a pictograph be used to answer questions involving predictions and in what ways are variation and uncertainty acknowledged in the language associated with making predictions? Does this differ with grade?
- 2. Does overall performance of students in relation to pictograph tasks differ across grades, and if so how?
- 3. What is the reaction of students across the grades to questions based on pictographs?

The implications of the outcomes for these questions are the focus of the Discussion.

Methodology

Task. Students answered the task in Figure 2 as part of a larger chance and data survey, which included questions related to chance, sampling, graphing, average, inference, and variation (Watson, Kelly, Callingham, & Shaughnessy, 2003). The task was developed based on previous research using an interview protocol on the same topic based on a bar graph (Watson & Moritz, 1999). All six parts of the task focus attention on the pictograph, with Parts a) and b) setting the scene by graph reading, Part c) associated with variation, and Parts d), e), and f) including inference through prediction. The format of the task reflected the need to provide questions to engage younger students, in particular, the first two parts. The other four parts, however, were open-ended and required written explanations. Literacy was hence required to read and interpret the task, and then to compose a response. Variations to the task in Figure 2 compared to the interview task used in earlier research include the representation of a larger number of students, 27 rather than 18; the presence of a category with no frequency data, the train; and the distinction of boys and girls with icons, adding another variable to the task.

Bus	E	窬	魯	8	8	灂	窗	灂	窗	8	
Car	₽	感	際日	8	際日	盛日					
Walk		8	8	Ô	8	8 11	8	8			
Train											
Bike	dip	8	\$	\$	\$	\$	8				
		1	2	3	4	5	б	7	8	9	10
					Number	of student	S				
) Hov	w many cl	nildren wa	lk to sch	ool?							
	w many m	ore child									
						why not					

f) Tom is not at school today. How do you think he will get to school tomorrow? Why?

Figure 2. Travel graph and survey questions.

Sample. The sample consisted of 730 students from grades 3 (n = 172), 5 (n = 183), 7 (n = 184), and 9 (n = 191) in nine public schools in the Australian state of Tasmania. The schools were chosen to reflect a range of socio-economic backgrounds. The authors administered the survey to the students as a class with their teacher present, assisting with the reading of questions where necessary.

Analysis. For this study the basic graph reading parts, a) and b), were coded on a correct-incorrect basis only. For Part a) a code of 1 was given to a response of "7", and for Part b) to a response of "4". Responses to the other four parts were categorized and similar responses clustered together into separate codes based on the type of reasoning shown (Miles & Huberman, 1994). Criteria were associated with increasing statistical

appropriateness and the use of language associated with variation and uncertainty. Explanations of the codes, with examples, are given in the results.

In terms of the specific research questions, the first is considered through the coding scheme developed and examples given, as well as the distribution of responses for the four grades. The second is answered for the average Total score of the four grades on the task using *t*-tests and the third research question is answered with anecdotal evidence from the teachers involved and from the authors.

Results

Research Question 1. Table 1 contains examples of the different types of responses by students to Parts c) to f) as a basis for discussion of the second research question. For Part c) the majority of students showed an appreciation of variation with 70% of students overall using language such as "might" or "could" that indicated the possibility of change from day to day in the pictograph (code 2). The grade 3 students experienced slightly more difficulty, with 55% explicitly acknowledging change, whereas 81% of the grade 9 students did so. In contrast to Part c), which encouraged language of possibility, Part e) about the row with the train, did not elicit responses acknowledging uncertainty about the meaning of the absence of data. Rather, 84% of students made declarative statements about the context based on direct interpretations or story telling (code 1). Only grade 3 students made fewer such comments, with 73%, but 24% provided completely inappropriate responses (code 0). Overall, only about 4% of students included uncertainty in the language of their responses to Part e) (code 2).

Parts d) and f) were considered similar in nature by the authors and thus coded in a parallel hierarchical fashion, with the lowest two codes for responses not engaging the graph, the middle two using the graph but inappropriately in a statistical sense, and the highest two using the graph in a statistically appropriate fashion (see Table 1 for examples). The two parts provide, however, an interesting contrast in terms of the distributions of responses over the different codes in Table 1.

Figure 3 shows the percent in each category for each grade graphically for Parts d) and f). Although the percent of code 0 responses was similar for the two parts, for Part f), the most common response for all grades did not involve using information from the graph (code 1), with 65% overall saying there was not enough information, giving a prediction but without an explanation, or inventing a story. The percent in this category for Part f) did, however, decrease with increasing grade. On the other hand, for Part d), this same category of response attracted only 30% of responses overall, with no grade trend. Over half of grade 3 students responded in this category but only 17% of grade 5 students, 22% of grade 7 students, and 30% of grade 9 students said there was not enough information or made up a story.

Codes 2 and 3 used information from the graph but not in a statistical sense. For Part d), pattern recognition (code 2) was the most common response overall (33%). This question may have drawn attention to the patterns of boys and girls in the graph and in particular to the row with the car. Looking for a pattern was not as common, however, for Part f). Only 8% of students overall suggested that Tom's mode of transport "could be any," or based their predictions on a pattern in the graph (e.g., car). The idea of balancing

the information in the graph (code 3) by selecting "boy" in Part d) to make the class have an equal number of each gender was more common than selecting "train" in Part f) to provide data in that row. For both parts, grade 5 students chose a balancing approach more often than other grades. Code 3 responses with balancing were considered more advanced than code 2 responses with pattern recognition because in considering frequency they were in transition to a statistically appropriate approach.

Table 1.

Code	Summary	Examples			
c) Would	d the graph look the same everyday?	Why or why not?			
2	Realistic recognition of variation	No, sometimes people are sick.			
	6	No, you <i>might</i> do something different			
1	Implicit recognition of variation	No, because people do different things on different days			
	(no uncertainty)	No, because you change			
0	Inappropriate responses	Yes, some things just don't change (no variation)			
		No, that's what I think (no reasoning)			
		Yes, because their legs would hurt (idiosyncratic)			
		I don't know (No response)			
d) A nev	v student came to school by car. Is the	he new student a boy or a girl? How do you know?			
5	Statistical reasons with	<i>Probably</i> a girl, more girls get a car to school			
5	uncertainty (variation)	Girl, You don't [know], it is just more likely to be a gi			
		because there are more			
4	Statistical reasons (no	Girl, because the graph shows the majority are girls			
-	uncertainty)	Girl, because there are 4 girls and only one boy			
3	Balancing	Boy, cause it's the only boy that goes by car			
	6	Boy, it could make 14 of both in the class			
2	Patterns and Anything can	Boy, because there is a pattern			
	happen	Girl, because she is at the end			
	11	It could be either, boy or girl			
1	No graph interaction	I'm not sure, because it doesn't have enough information			
	0 1	Boy, I just guessed			
0	Inappropriate responses	There were more kids (Idiosyncratic)			
		I don't know (No response)			
e) What	does the row with the train tell about	t how children get to school?			
2	Acknowledging variation and	No one catches the train on this day			
	uncertainty	They <i>might</i> catch the train			
1	Direct graph reading and	They don't catch the train			
	interpretation	There are no trains			
	I I I I I I I I I I I I I I I I I I I	Some people don't like trains			
0	Inappropriate responses	Nothing (Ambiguous)			
-		A train station (Idiosyncratic)			
		Pass (No response)			
f) Tom i	s not at school today. How do you t	hink he will get to school tomorrow? Why?			
5	Statistical reasons with	<i>Probably</i> by bus, because 1/3 of the children caught it			
U	uncertainty (variation)	today			
	uncertainty (variation)	<i>Most likely</i> by bike, because most of the boys come by bik			
4	Statistical reasons (no	Bike, the majority of boys ride to school			
	uncertainty)	Bus, more people catch the bus			
3	Balancing	Train, because there is no one on the train today			
	c				
2	Patterns and Anything can	Car or bus, the pattern			
	happen	Anything, chance!			

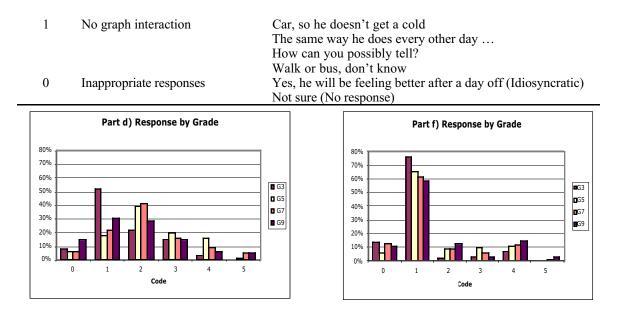


Figure 3. Percent of response by grade for each code of Part d) and Part f).

Overall, the percent making a decision based on appropriate statistical reasoning (e.g., "more girls come by car" for Part d) and "more kids come by bus" for Part f)), without acknowledging uncertainty (code 4), was similar for both Parts d) and f). Again it was most common for grade 5 students in Part d), but for Part f), the percent increased with grade. The acknowledgement of uncertainty along with a statistically appropriate choice (code 5) was uncommon for all grades, peaking at about 5% for grades 7 and 9 on Part d), and dropping to about 2% on Part f). Overall, 3.7% of students responded with a code 4 or 5 to both Parts d) and f) by using statistically appropriate approaches.

Research Question 2. For the second research question, Table 2 contains mean Total scores and standard deviations for each grade. As can be seen there is little change among the top three grades on the task. There are, however, statistically significant differences between grades 3 and 5 (t = -8.33, p < .001), grades 3 and 7 (t = -6.28, p < .001), and grades 3 and 9 (t = -6.79, p < .001) on the mean Total Score. The differences in average scores for the individual parts of the question were either non-existent or reflected the overall difference involving grade 3. A ceiling effect for the basic graph reading parts was evident over the higher grades.

Table 2.Mean Total Score and Standard Deviations for Each Grade

	Grade 3	Grade 5	Grade 7	Grade 9
Mean	6.30	8.20	7.85	7.95
Std Dev	2.28	2.01	2.37	2.34
n	172	183	184	191

Out of a total possible score of 16 for the task, the average score only approached half of that, reflecting the performance shown in Figure 3. The range of scores was 0 to 16, with eight students in grades 7 and 9 scoring 14 or more and the best overall responses by a grade 7 student who answered correctly to Parts a) and b), and who acknowledged variation

in Part c) ("No, One might be sick or changing their plans") and Part e) ("No children catch a train to school for that day"), and responded with uncertainty when using statistically appropriate methods when predicting in Parts d) ("It doesn't show you but in the graph, more than likely a girl because there's 4, when there's 1 boy") and Part f) ("Don't know but using the graph, it might be bus"). Although grade 5 students had a higher mean Total score the highest overall Total score obtained by a grade 5 student was 13.

Research Question 3. With respect to the question of the reaction of students to the task, anecdotal evidence from conversations with the teachers after the survey suggests that this was one of the most difficult questions for students to answer on the survey. Even some of the teachers themselves admitted to feeling uncomfortable with this question since there were no "correct" answers to Parts c) through f). The existence of statistically appropriate responses was not always obvious to teachers and was a matter of discussion and debate in debriefing sessions. The authors found that when administering the survey, there were more queries about this question from students than from any other question on the survey. Many students had a strong desire to know whether they had the answer correct. This occurred not only at the primary grades but also in high school.

Discussion

The results of this study support those of others who found pattern observation, the use of topic information, and refusal to predict due to the perceived lack of information as significant distracters in graph interpretation and prediction (Pereira-Mendoza & Mellor, 1991; Pereira-Mendoza et al., 1995; Watson & Kelly, 2002). Although all of the four openended parts to the task were based on the same pictograph, they elicited different levels of recognition of variation and uncertainty when prediction was involved. Most students are aware of variation from their life experiences and acknowledge it when specifically asked in Part c). They are not, however, apt to acknowledge variation spontaneously in explanations for missing data, for example, in the row with the train.

The prevalence of ideas associated with patterns and balancing, especially for Part d) (nearly 50% overall), but also to some extent for Part f) (13% overall), points to likely interference with other learning objectives in the mathematics curriculum, particularly early work with pattern recognition. Ideas of fairness may also be associated with Part d) in terms of balancing boys and girls. Context may play a further role in the rates of response suggesting pattern recognition and balancing. Part f) may be a more familiar context for students in terms of personal experience (Tom is not at school today) than Part d) (a new student arrives by car). For Part d), the question, however, points students to the row with the car where a pattern is more prominent. In Part f), the cue (Tom) is more implicit and finding a pattern or something to balance requires consideration of the whole graph rather than a single row. In this case using the topic information to tell a story based on life experiences is a more common response. Garfield and delMas (1991) in their study of students' conceptions of probability found that the contextual content of questions influenced the type of strategies used by students. Students did not appear to realise that they could use the same strategy to solve similar problems, which also seems to be the case in the present study.

Of those students who did use the information provided in the graph appropriately, about 13% for Part d) and 12% for Part f), many fewer included an element of uncertainty in the response. This may, to some extent be an artefact of the questions being specific, but may also reflect the outcome approach to prediction identified by Konold (1989).

The plateau in performance from grade 5 may reflect the lack of continued exposure to this type of graph in the middle school years and perhaps the consequent belief that pictographs are not important, are intended only "for little kids", and are not to be taken seriously. Combined with the observation of students' reluctance to answer questions with no certain correct answers, it is the authors' belief that pictographs should continue to attract attention through the middle years of schooling.

Graphs such as the one in Figure 1 show that pictographs can be important, are for adults as well as "kids", and are to be taken seriously. Modelling the communication of understanding of such graphical information, as suggested by Gal (2002) in his description of adult statistical literacy, should be encouraged by teachers, particularly in terms of the appropriate language of uncertainty to be used in making predictions. Comments made about this task by teachers in this study give an indication that adults also may be uncomfortable with this type of questioning due to lack of clear-cut, right-wrong responses. If necessary, teachers should acknowledge this discomfort in classroom discussions. Teachers also need to value student responses based on patterns, balancing, past experiences (story telling), and the perception there is not enough information, but contrast them on one hand with expectations of the mathematics curriculum and on the other hand with appropriate statistical considerations. Otherwise, these views may follow students into adulthood.

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